Unit 3

Sequences & Series

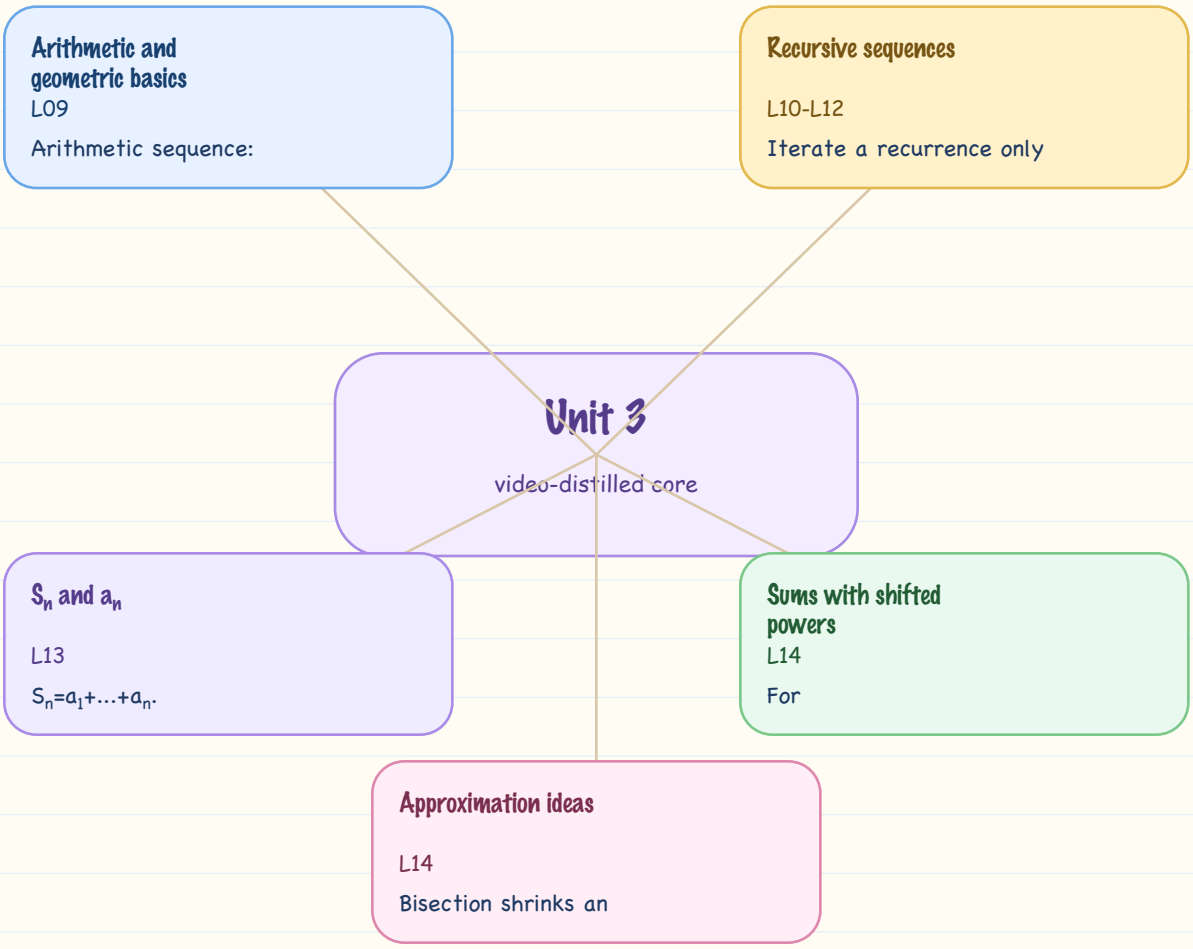
SOURCE LOCK

6 distilled lessons

RULE FOR THIS PREVIEW

Only translated/condensed material from the video-distilled lesson notes is used.

Unit map



Teacher Moves

how to attack problems

Strategy moves distilled from the teacher's worked explanations.

MOVE 1

Count before formula

L09

- Count the number of terms before deciding the exponent or final index.
- This prevents off-by-one errors in sequence formulas.

MOVE 2

Avoid brute force

L09

- Use equal-index sums in arithmetic sequences and equal-index products in geometric sequences.
- This is the shortcut for avoiding term-by-term calculation.

MOVE 3

Classify the recurrence

L10

- Ask whether the recurrence is additive, multiplicative, or a mixed form needing transformation.
- Then choose summing, multiplying, or constructing an auxiliary sequence.

Teacher Moves

MOVE 4

Check small n

L10

- Use n=1 and n=2 to test whether the formula matches the first term and recurrence.

MOVE 5

Shift to a fixed point

L11

- For $a_{n+1}=p\ a_n+r$, look for the fixed point L.
- Then study $b_n=a_n-L$ as a geometric sequence.

MOVE 6

Write one more line

L12-L13

- When a sum relation is long, write the n+1 version and subtract.
- This compresses many terms into neighboring terms.

MOVE 7

Shifted subtraction warning

L14

- Before using shifted subtraction, check whether the common ratio q equals 1.
- If q=1, switch methods instead of dividing by 1-q.

Teacher Moves

MOVE 8 — Structure before formula — L09

- There are not many sequence formulas.
- The hard part is recognizing recurrence, index, and summation structure.

MOVE 9 — Block sums are sequences — L09

- For equal-length consecutive blocks, the block sums may preserve arithmetic structure.
- Use S_m and S_{2m} by turning them into adjacent block sums.

MOVE 10 — Check new geometric blocks — L09

- Sums or products of neighboring geometric terms can form a new geometric sequence.
- Check that the new terms are not zero.

MOVE 11 — Choose by operation — L11

- Choose the recurrence method by the operation structure.
- Do not choose by problem number or surface length.

Teacher Moves

MOVE 12

Index range discipline

L11

- When stacking or multiplying recurrence lines, keep the index range unified.
- The last line should connect exactly to a_n.

MOVE 13

Telescoping edges survive

L11

- In telescoping sums, middle terms cancel but boundary terms stay.
- Write the first and last uncancelled terms before simplifying.

MOVE 14

Start-index discipline

L14

- Before summing kx^k, decide whether the index starts at $k=0$ or $k=1$.
- Changing the start index changes the boundary terms.

MOVE 15

Closed-form sanity check

L14

- After deriving a closed form, test it with a small n.
- This catches off-by-one and missing-boundary mistakes.

Cornell Notes

Arithmetic and geometric basics

cue

Arithmetic and geometric basics

L09

- Arithmetic sequence: $a_n=a_1+(n-1)d$.
- Arithmetic sum: $S_n=n(a_1+a_n)/2=n[2a_1+(n-1)d]/2$.
- Geometric sequence: $a_n=a_1q^{n-1}$.
- If q is not 1, geometric sum: $S_n=a_1(1-q^n)/(1-q)$. If $q=1$, $S_n=na_1$.
- Equal-index sums work in arithmetic sequences; equal-index products work in geometric sequences.

Recursive sequences

cue

Recursive sequences

L10-L12

- Iterate a recurrence only when the pattern stays readable.
- Difference recurrences are handled by summing; ratio recurrences are handled by multiplying.
- For $a_{n+1}=pa_n+r$, look for the fixed point $L=r/(1-p)$ when p is not 1.
- Set $b_n=a_n-L$ to turn the recurrence into a geometric one.
- For nonlinear recurrences, try completing the square, taking logs, or splitting odd and even terms.

S_n and a_n

cue

S_n and a_n

L13

- $S_n=a_1+\ldots+a_n$.
- For $n\geq 2$, $a_n=S_n-S_{n-1}$.
- Keep $a_1=S_1$ separate when using the difference formula.
- If a recurrence is given for S_n, solve S_n first, then recover a_n.

Cornell Notes

cue

Sums with shifted powers

L14

Sums with shifted powers

- For arithmetic-times-geometric sums, write T_n and qT_n, then subtract.
- Boundary terms do not cancel; write them carefully.
- If q=1, do not divide by 1-q; use an arithmetic-sum approach instead.
- Weighted power sums such as sum kx^k use the same shifted-subtraction idea.

cue

Approximation ideas

L14

Approximation ideas

- Bisection shrinks an interval where the sign changes.
- Newton's method uses the tangent line at x_k to make the next approximation.
- Newton update: $x_{k+1}=x_k-f(x_k)/f'(x_k)$.

Worked Example Cards

video example card L09 example

Arithmetic index property

1. If $\{a_n\}$ is arithmetic and $1+8=4+5$, the same total difference is crossed.
2. Therefore $a_1+a_8=a_4+a_5$.

video example card L10 example

Product telescoping recurrence

1. From $(n+1)a_{n+1}=na_n$, get $a_{n+1}/a_n=n/(n+1)$.
2. Multiplying from 1 to n-1 cancels middle factors.
3. With $a_1=1$, $a_n=1/n$.

video example card L13 example

From S_n to a_n

1. If $S_n=2n^2-5n$, first $a_1=S_1=-3$.
2. For $n>=2$, compute S_n-S_{n-1}.
3. The result is $a_n=4n-7$, and it also works for $n=1$.