Unit 3

Sequences & Series

SOURCE

6 video lessons

SOURCE LOCK

Source-locked summary: translated and condensed only from Lessons 9-14 distilled video notes.
No added practice problems or outside examples are included.
Every section below carries a lesson/source label.

Arithmetic and geometric basics

L09

- Arithmetic sequence: $a_n=a_1+(n-1)d$.
- Arithmetic sum: $S_n=n(a_1+a_n)/2=n[2a_1+(n-1)d]/2$.
- Geometric sequence: $a_n=a_1q^{n-1}$.
- If q is not 1, geometric sum: $S_n=a_1(1-q^n)/(1-q)$. If q=1, $S_n=na_1$.
- Equal-index sums work in arithmetic sequences; equal-index products work in geometric sequences.

Recursive sequences

L10-L12

- Iterate a recurrence only when the pattern stays readable.
- Difference recurrences are handled by summing; ratio recurrences are handled by multiplying.
- For $a_{n+1}=pa_n+r$, look for the fixed point $L=r/(1-p)$ when p is not 1.
- Set $b_n=a_n-L$ to turn the recurrence into a geometric one.
- For nonlinear recurrences, try completing the square, taking logs, or splitting odd and even terms.

S_n and a_n

L13

- $S_n=a_1+...+a_n$.
- For $n>=2$, $a_n=S_n-S_{n-1}$.
- Keep $a_1=S_1$ separate when using the difference formula.

- If a recurrence is given for S_n, solve S_n first, then recover a_n.

Sums with shifted powers

L14

- For arithmetic-times-geometric sums, write T_n and qT_n, then subtract.
- Boundary terms do not cancel; write them carefully.
- If q=1, do not divide by 1-q; use an arithmetic-sum approach instead.
- Weighted power sums such as sum kx^k use the same shifted-subtraction idea.

Approximation ideas

L14

- Bisection shrinks an interval where the sign changes.
- Newton's method uses the tangent line at x_k to make the next approximation.
- Newton update: $x_{k+1}=x_k-f(x_k)/f'(x_k)$.

teacher tips from class

COUNT BEFORE FORMULA · L09

Count the number of terms before deciding the exponent or final index.
This prevents off-by-one errors in sequence formulas.

AVOID BRUTE FORCE · L09

Use equal-index sums in arithmetic sequences and equal-index products in geometric sequences.
This is the shortcut for avoiding term-by-term calculation.

CLASSIFY THE RECURRENCE · L10

Ask whether the recurrence is additive, multiplicative, or a mixed form needing transformation.
Then choose summing, multiplying, or constructing an auxiliary sequence.

CHECK SMALL N · L10

Use n=1 and n=2 to test whether the formula matches the first term and recurrence.

SHIFT TO A FIXED POINT · L11

For $a_{n+1}=p\ a_n+r$, look for the fixed point L.
Then study $b_n=a_n-L$ as a geometric sequence.

WRITE ONE MORE LINE · L12-L13

When a sum relation is long, write the n+1 version and subtract.
This compresses many terms into neighboring terms.

SHIFTED SUBTRACTION WARNING · L14

Before using shifted subtraction, check whether the common ratio q equals 1.
If q=1, switch methods instead of dividing by 1-q.

STRUCTURE BEFORE FORMULA · L09

There are not many sequence formulas.
The hard part is recognizing recurrence, index, and summation structure.

BLOCK SUMS ARE SEQUENCES · L09

For equal-length consecutive blocks, the block sums may preserve arithmetic structure.
Use S_m and S_2m by turning them into adjacent block sums.

CHECK NEW GEOMETRIC BLOCKS · L09

Sums or products of neighboring geometric terms can form a new geometric sequence.
Check that the new terms are not zero.

CHOOSE BY OPERATION · L11

Choose the recurrence method by the operation structure.
Do not choose by problem number or surface length.

INDEX RANGE DISCIPLINE · L11

When stacking or multiplying recurrence lines, keep the index range unified.
The last line should connect exactly to a_n.

TELESCOPING EDGES SURVIVE · L11

In telescoping sums, middle terms cancel but boundary terms stay.
Write the first and last uncancelled terms before simplifying.

START-INDEX DISCIPLINE · L14

Before summing kx^k, decide whether the index starts at k=0 or k=1.
Changing the start index changes the boundary terms.

CLOSED-FORM SANITY CHECK · L14

After deriving a closed form, test it with a small n.
This catches off-by-one and missing-boundary mistakes.

video example

L09 example

PROBLEM

Arithmetic index property

Step 1: If $\{a_n\}$ is arithmetic and 1+8=4+5, the same total difference is crossed.
Step 2: Therefore $a_1+a_8=a_4+a_5$.

video example

L10 example

PROBLEM

Product telescoping recurrence

Step 1: From $(n+1)a_{n+1}=na_n$, get $a_{n+1}/a_n=n/(n+1)$.

Step 2: Multiplying from 1 to n-1 cancels middle factors.

Step 3: With $a_1=1$, $a_n=1/n$.

video example

L13 example

PROBLEM

From S_n to a_n

Step 1: If $S_n=2n^2-5n$, first $a_1=S_1=-3$.

Step 2: For n>=2, compute S_n-S_{n-1}.

Step 3: The result is $a_n=4n-7$, and it also works for n=1.