## Unit 4

# Trigonometric Functions

**SOURCE LOCK**

5 distilled lessons

**RULE FOR THIS PREVIEW**

Only translated/condensed material from the video-distilled lesson notes is used.

## Unit map

**Unit 4**

video-distilled core

**Period, parity, and angle signs**
L15
A period T is a nonzero

**Six trig functions and identities**
L16
tan θ=sin θ/cos

**Graphs and transformations**
L16-L17
sin x is odd, cos x is

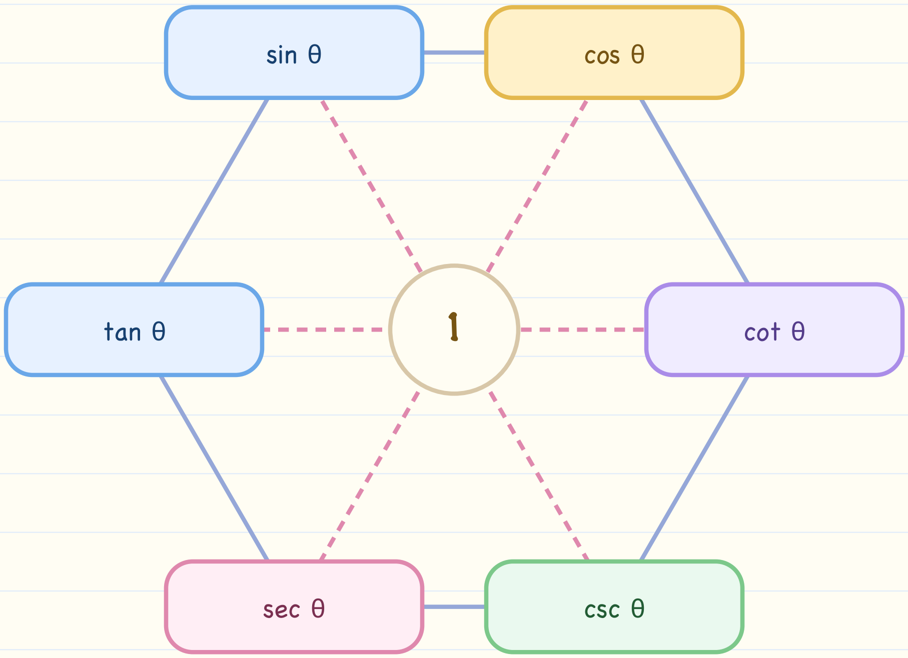
**Angle formulas and auxiliary angle**
L17-L18
sin(α +/- β)=sin

**Triangles**
L19
A+B+C=π in a triangle.

# Trig Hexagon

**Same-Angle Identity Hexagon** L16 02:01-05:40

Teacher's memory map for same-angle identities.




**how to read it**

- Same-angle identities are grouped as reciprocal, product, and square relations.
- Opposite vertices are reciprocal pairs: sin/csc, cos/sec, tan/cot.
- Neighbor products give the middle relation; keep tan=sin/cos and cot=cos/sin.
- Downward triangles give square identities: $\sin^2+\cos^2=1$, $\tan^2+1=\sec^2$, $1+\cot^2=\csc^2$.

# Teacher Moves

### how to attack problems

Strategy moves distilled from the teacher's worked explanations.

**MOVE 1** — Reduce, then decide quadrant — L15

- Move the angle into one standard cycle before deciding the quadrant.
- Reference angle gives the size; quadrant gives the sign.

**MOVE 2** — Hexagon identity map — L16

- Use the teacher's hexagon to sort identities into reciprocal, product, and square relations.
- Read opposite vertices, neighboring products, and downward triangles instead of memorizing a loose list.

**MOVE 3** — Triangle from tangent — L16

- If tan is known, draw a right triangle from opposite/adjacent sides.
- Use Pythagoras for the hypotenuse, then attach signs from the quadrant.

# Teacher Moves

**MOVE 4**

### Induction formula routine

L16

- Rewrite the angle as k*π/2 +/- α.
- Odd multiples of π/2 swap the function name; the quadrant gives the final sign.

**MOVE 5**

### Graph from the base wave

L16-L17

- For complicated periods or transformations, sketch the base graph first.
- Then apply the transformation to the whole phase.

**MOVE 6**

### Solve the phase

L17

- For monotonic intervals or extrema of y=A sin(ω x+φ), first write the standard interval for sin u.
- Then solve u=ω x+φ.

**MOVE 7**

### Identity direction

L18-L19

- First unify the angle and function names.
- Then decide whether to use sum-difference, double-angle, power-reduction, or auxiliary-angle form.

# Teacher Moves

**MOVE 8**

### Tangent graph order

L19

- Mark tangent asymptotes first.
- Then draw the increasing branches through the zeros.

**MOVE 9**

### Treat the whole phase

L17

- Horizontal movement acts on the whole inside expression.
- Handle $\omega$ first, then the phase shift.

**MOVE 10**

### Negative amplitude swap

L17

- If $A<0$ in $A \sin u$, maximum and minimum positions switch.
- Do not reuse the $A>0$ phase rule blindly.

**MOVE 11**

### Phase from evidence

L18

- Use max/min values for amplitude and midline.
- Then use a known point or starting direction to determine phase.

# Teacher Moves

**MOVE 12** — Universal substitution warning — L18

- When using t=tan(x/2), record angles where the substitution is undefined.
- Also check whether any original angles were lost.

**MOVE 13** — Special-angle elimination — L18

- For multiple choice, plug in special angles to eliminate options.
- Avoid values that make the expression undefined.

**MOVE 14** — Do not reduce blindly — L19

- After reducing $\sin^2 x$ or $\cos^2 x$ to cos 2x, check whether auxiliary angle form is still useful.
- Sometimes keeping a same-name, same-angle structure is better.

**MOVE 15** — Minus sign changes the story — L19

- When you factor out a negative sign, adjust the phase or function sign with it.
- Otherwise max/min conclusions can reverse.

# Teacher Moves

**MOVE 16**

AM-GM equality check

L19

- AM-GM can give a bound for positive expressions.
- The equality case must satisfy both the domain and the variable range.

# Cornell Notes

cue

Period, parity, and angle signs

L15

## Period, parity, and angle signs

- A period T is a nonzero number with $f(x+T)=f(x)$ on the domain.
- Not every periodic function has a smallest positive period.
- Even functions have y-axis symmetry. Odd functions have origin symmetry.
- For trig signs: sin uses y-coordinate, cos uses x-coordinate, tan=sin/cos.
- QI all positive; QII sin positive; QIII tan positive; QIV cos positive.

cue

Six trig functions and identities

L16

## Six trig functions and identities

- $\tan\theta=\sin\theta/\cos\theta$, and $\cot\theta=\cos\theta/\sin\theta$.
- $\sec\theta=1/\cos\theta$, and $\csc\theta=1/\sin\theta$.
- The teacher organizes same-angle identities with a Bagua / regular hexagon map.
- Opposite vertices give reciprocal pairs; neighboring products give the middle relation.
- Downward triangles encode the square identities.
- $\sin^2\theta+\cos^2\theta=1$.
- $1+\tan^2\theta=\sec^2\theta$; $1+\cot^2\theta=\csc^2\theta$.
- Use quadrant information to attach the correct sign.

cue

Graphs and transformations

L16-L17

## Graphs and transformations

- sin x is odd, cos x is even, and both have basic period $2\pi$.
- $|\sin x|$, $\sin^2 x$, and $\cos^2 x$ have period $\pi$.
- For $y=A\sin(\omega x+\varphi)+C$, amplitude is $|A|$.
- If $\omega$ is not 0, period $T=2\pi/|\omega|$.
- Zeros, extrema, symmetry axes, and monotonic intervals come from solving the inner phase.

# Cornell Notes

cue

Angle formulas and auxiliary angle

L17-L18

## Angle formulas and auxiliary angle

- sin(α +/- β)=sin α cos β +/- cos α sin β.
- cos(α +/- β)=cos α cos β minus-or-plus sin α sin β.
- sin 2α=2sin α cos α.
- cos 2α=$\cos^2$ α−$\sin^2$ α=1−2$\sin^2$ α=2$\cos^2$ α−1.
- A sin u+B cos u=R sin(u+φ), with R=sqrt($A^2$+$B^2$).

cue

Triangles

L19

## Triangles

- A+B+C=π in a triangle.
- Sine law: a/sin A=b/sin B=c/sin C=2R.
- Cosine law: $a^2$=$b^2$+$c^2$−2bc cos A.
- Area: S=(1/2)bc sin A.
- Use side-angle information to choose sine law, cosine law, or area formula.

# Worked Example Cards

**video example card** L15 example

sin(2π/3)

1. 2π/3 is in Quadrant II, so sine is positive.
2. The reference angle is π/3.
3. sin(2π/3)=sqrt(3)/2.

**video example card** L16 example

Given tan α=5/12 in Quadrant III

1. Use the 5-12-13 right triangle for absolute values.
2. In Quadrant III, sin and cos are both negative.
3. sin α=-5/13, cos α=-12/13.

**video example card** L17 example

Find amplitude and period of y=3sin(2x+π/3)-1

1. Amplitude is |3|=3.
2. Period is 2π/|2|=π.

**video example card** L19 example

Cosine law with b=5, c=7, A=60 degrees

1. $a^2=b^2+c^2-2bc\cos A$.
2. cos 60 degrees=1/2.
3. $a^2=25+49-35=39$.