

Unit 4

Trigonometric Functions

SOURCE

5 video lessons

SOURCE LOCK

Source-locked summary: translated and condensed only from Lessons 15-19 distilled video notes.

No added practice problems or outside examples are included.

Every section below carries a lesson/source label.

Period, parity, and angle signs

L15

- A period T is a nonzero number with $f(x+T)=f(x)$ on the domain.
- Not every periodic function has a smallest positive period.
- Even functions have y -axis symmetry. Odd functions have origin symmetry.
- For trig signs: \sin uses y -coordinate, \cos uses x -coordinate, $\tan=\sin/\cos$.
- QI all positive; QII \sin positive; QIII \tan positive; QIV \cos positive.

Six trig functions and identities

L16

- $\tan \theta = \sin \theta / \cos \theta$, and $\cot \theta = \cos \theta / \sin \theta$.
- $\sec \theta = 1 / \cos \theta$, and $\csc \theta = 1 / \sin \theta$.
- The teacher organizes same-angle identities with a Bagua / regular hexagon map.
- Opposite vertices give reciprocal pairs; neighboring products give the middle relation.
- Downward triangles encode the square identities.
- $\sin^2 \theta + \cos^2 \theta = 1$.
- $1 + \tan^2 \theta = \sec^2 \theta$; $1 + \cot^2 \theta = \csc^2 \theta$.
- Use quadrant information to attach the correct sign.

Graphs and transformations

L16-L17

- $\sin x$ is odd, $\cos x$ is even, and both have basic period 2π .

- $|\sin x|$, $\sin^2 x$, and $\cos^2 x$ have period π .
- For $y=A \sin(\omega x+\phi)+C$, amplitude is $|A|$.
- If ω is not 0, period $T=2\pi/|\omega|$.
- Zeros, extrema, symmetry axes, and monotonic intervals come from solving the inner phase.

Angle formulas and auxiliary angle

L17-L18

- $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$.
- $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$.
- $\sin 2\alpha = 2\sin \alpha \cos \alpha$.
- $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - 2\sin^2 \alpha = 2\cos^2 \alpha - 1$.
- $A \sin u + B \cos u = R \sin(u+\phi)$, with $R = \sqrt{A^2 + B^2}$.

Triangles

L19

- $A+B+C=\pi$ in a triangle.
- Sine law: $a/\sin A = b/\sin B = c/\sin C = 2R$.
- Cosine law: $a^2 = b^2 + c^2 - 2bc \cos A$.
- Area: $S = (1/2)bc \sin A$.
- Use side-angle information to choose sine law, cosine law, or area formula.

teacher tips from class

REDUCE, THEN DECIDE QUADRANT · L15

Move the angle into one standard cycle before deciding the quadrant.
Reference angle gives the size; quadrant gives the sign.

HEXAGON IDENTITY MAP · L16

Use the teacher's hexagon to sort identities into reciprocal, product, and square relations.
Read opposite vertices, neighboring products, and downward triangles instead of memorizing a loose list.

TRIANGLE FROM TANGENT · L16

If \tan is known, draw a right triangle from opposite/adjacent sides.
Use Pythagoras for the hypotenuse, then attach signs from the quadrant.

INDUCTION FORMULA ROUTINE · L16

Rewrite the angle as $k \cdot \pi/2 \pm \alpha$.
Odd multiples of $\pi/2$ swap the function name; the quadrant gives the final sign.

GRAPH FROM THE BASE WAVE · L16-L17

For complicated periods or transformations, sketch the base graph first.
Then apply the transformation to the whole phase.

SOLVE THE PHASE · L17

For monotonic intervals or extrema of $y=A \sin(\omega x + \phi)$, first write the standard interval for $\sin u$.
Then solve $u = \omega x + \phi$.

IDENTITY DIRECTION · L18-L19

First unify the angle and function names.
Then decide whether to use sum-difference, double-angle, power-reduction, or auxiliary-angle form.

TANGENT GRAPH ORDER · L19

Mark tangent asymptotes first.
Then draw the increasing branches through the zeros.

TREAT THE WHOLE PHASE · L17

Horizontal movement acts on the whole inside expression.
Handle ω first, then the phase shift.

NEGATIVE AMPLITUDE SWAP · L17

If $A < 0$ in $A \sin u$, maximum and minimum positions switch.
Do not reuse the $A > 0$ phase rule blindly.

PHASE FROM EVIDENCE · L18

Use max/min values for amplitude and midline.
Then use a known point or starting direction to determine phase.

UNIVERSAL SUBSTITUTION WARNING · L18

When using $t = \tan(x/2)$, record angles where the substitution is undefined.
Also check whether any original angles were lost.

SPECIAL-ANGLE ELIMINATION · L18

For multiple choice, plug in special angles to eliminate options.
Avoid values that make the expression undefined.

DO NOT REDUCE BLINDLY · L19

After reducing $\sin^2 x$ or $\cos^2 x$ to $\cos 2x$, check whether auxiliary angle form is still useful.
Sometimes keeping a same-name, same-angle structure is better.

MINUS SIGN CHANGES THE STORY · L19

When you factor out a negative sign, adjust the phase or function sign with it.
Otherwise max/min conclusions can reverse.

AM-GM EQUALITY CHECK · L19

AM-GM can give a bound for positive expressions.
The equality case must satisfy both the domain and the variable range.

video example

L15 example

PROBLEM

$$\sin(2\pi/3)$$

Step 1: $2\pi/3$ is in Quadrant II, so sine is positive.

Step 2: The reference angle is $\pi/3$.

Step 3: $\sin(2\pi/3) = \sqrt{3}/2$.

video example

L16 example

PROBLEM

Given $\tan \alpha = 5/12$ in Quadrant III

Step 1: Use the 5-12-13 right triangle for absolute values.

Step 2: In Quadrant III, sin and cos are both negative.

Step 3: $\sin \alpha = -5/13$, $\cos \alpha = -12/13$.

video example

L17 example

PROBLEM

Find amplitude and period of $y = 3\sin(2x + \pi/3) - 1$

Step 1: Amplitude is $|3| = 3$.

Step 2: Period is $2\pi/|2| = \pi$.

video example

L19 example

PROBLEM

Cosine law with $b=5$, $c=7$, $A=60$ degrees

Step 1: $a^2 = b^2 + c^2 - 2bc \cos A$.

Step 2: $\cos 60 \text{ degrees} = 1/2$.

Step 3: $a^2 = 25 + 49 - 35 = 39$.