

Unit 1

Functions: Representation & Properties

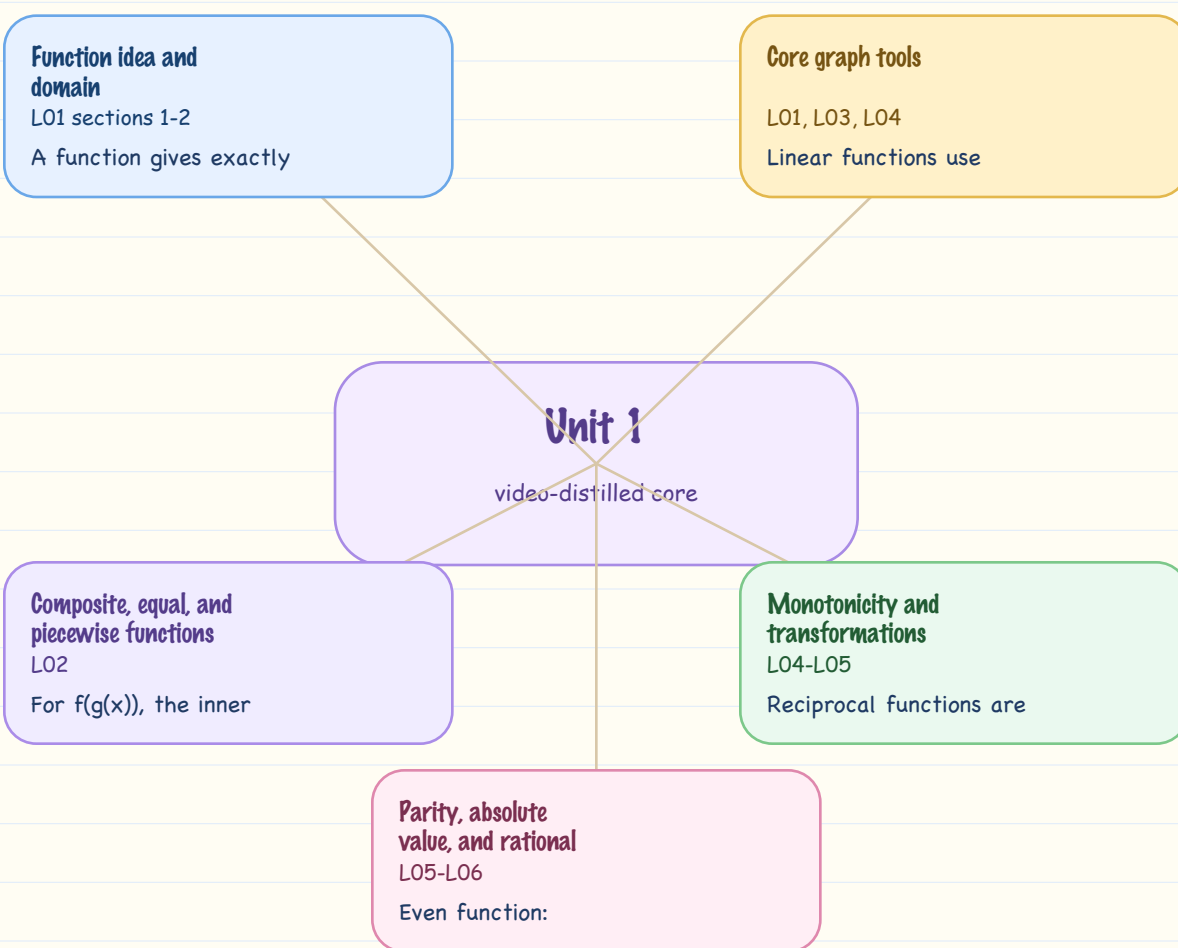
SOURCE LOCK

6 distilled lessons

RULE FOR THIS PREVIEW

Only translated/condensed material from the video-distilled lesson notes is used.

Unit map



Teacher Moves

how to attack problems

Strategy moves distilled from the teacher's worked explanations.

Domain first, simplify second

L01

MOVE 1

- Before simplifying an expression, write the original restrictions.
- Do not let cancellation accidentally enlarge the domain.

Range from pieces

L03

MOVE 2

- For a piecewise range question, sketch each interval piece separately.
- Then merge all possible y-values.

Quadratic interval routine

L04

MOVE 3

- First read the opening direction.
- Then compare the axis of symmetry with the interval endpoints.

Teacher Moves

Parity routine

L05-L06

MOVE 4

- Check whether the domain is symmetric about 0 before testing $f(-x)$.
- For absolute value or piecewise functions, decide the branch first.

Rational graph routine

L06

MOVE 5

- Find the two asymptotes and one known point first.
- Exclude degenerate constant cases before claiming monotonic behavior.

Choose the quadratic form

L01

MOVE 6

- Use factored form when zeros are given.
- Use vertex form when the vertex or axis is given.

Endpoint notation check

L01

MOVE 7

- Use square brackets only when an endpoint is actually included.
- Infinity never gets a square bracket.

Teacher Moves

Parameter degeneration first

L01

MOVE 8

- In parameter quadratic problems, first check whether the quadratic term vanishes.
- Only after that discuss opening direction, axis, and extrema.

Discrete input stays discrete

L02

MOVE 9

- If the domain is a finite or discrete set, graph isolated points.
- Do not connect them just because the formula looks linear.

Bounds must be reached

L03

MOVE 10

- A maximum or minimum is not just a bound.
- The function must actually attain that value.

Do not cross a break

L04

MOVE 11

- A monotonic interval cannot cross a point where the function is undefined.
- Split intervals at holes, asymptotes, and branch changes.

Teacher Moves

$ax+b/x$ shape read

L06

MOVE 12

- The sign of ab decides the double-hook or double-slash shape.
- Turning points occur when the two absolute-value parts are equal.

Cornell Notes

cue

Function idea and domain

L01 sections 1-2

Function idea and domain

- A function gives exactly one output for every allowed input.
- A graph passes the vertical-line test when it represents y as a function of x .
- Domain means allowed inputs. Range means possible outputs.
- Before simplifying, record the original domain.
- Restrictions: denominator not 0; even-root radicand ≥ 0 ; all restrictions must overlap.

cue

Core graph tools

L01, L03, L04

Core graph tools

- Linear functions use slope to show increasing or decreasing direction.
- Quadratic features come from axis, vertex, opening direction, and interval endpoints.
- For a quadratic on a restricted interval, check the vertex and both endpoints.
- Real-world formulas need geometric or physical domain restrictions, not only algebra.

cue

Composite, equal, and piecewise functions

L02

Composite, equal, and piecewise functions

- For $f(g(x))$, the inner value $g(x)$ must land inside the domain of f .
- Two functions are equal only when their domains and rules are both the same.
- Letters are placeholders; domain and rule are what matter.
- Piecewise functions are evaluated from the correct interval first, then substituted.

Cornell Notes

cue

Monotonicity and transformations

L04-L05

Monotonicity and transformations

- Reciprocal functions are monotonic on each side of their break point, not across it.
- Difference quotients can prove increasing or decreasing behavior.
- Inside horizontal shifts act opposite the sign; outside vertical shifts act with the sign.
- For $f(x+c)$, pass the whole expression $x+c$ into the original interval.

cue

Parity, absolute value, and rational form

L05-L06

Parity, absolute value, and rational form

- Even function: $f(-x)=f(x)$, with y-axis symmetry.
- Odd function: $f(-x)=-f(x)$, with origin symmetry.
- For absolute-value or piecewise functions, decide which branch $-x$ belongs to before comparing.
- A non-degenerate linear fractional function has a vertical asymptote, horizontal asymptote, and center.

Worked Example Cards

video example card

L01 classroom example

Domain of $\sqrt{1-x} + \sqrt{x+3} - 1$

1. Need $1-x \geq 0$ and $x+3 \geq 0$.
2. So $x \leq 1$ and $x \geq -3$.
3. Domain: $[-3, 1]$.

video example card

L02 example

If f has domain $[0,1]$, find the domain of $f(\sqrt{x}-2)$

1. Require $\sqrt{x}-2$ in $[0,1]$.
2. So $0 \leq \sqrt{x}-2 \leq 1$.
3. Then $2 \leq \sqrt{x} \leq 3$, giving $4 \leq x \leq 9$.

video example card

L06 example

Show $f(x) = x|x| + px$ is odd

1. Remove absolute value by branches.
2. For $x > 0$, compare $f(x)$ with $f(-x)$ on the negative branch.
3. The result is $f(-x) = -f(x)$, so the function is odd.