

## Unit 1

# Functions: Representation & Properties

### SOURCE

6 video lessons

### SOURCE LOCK

Source-locked summary: translated and condensed only from Lessons 1-6 distilled video notes.

No added practice problems or outside examples are included.

Every section below carries a lesson/source label.

## Function idea and domain

L01 sections 1-2

- A function gives exactly one output for every allowed input.
- A graph passes the vertical-line test when it represents  $y$  as a function of  $x$ .
- Domain means allowed inputs. Range means possible outputs.
- Before simplifying, record the original domain.
- Restrictions: denominator not 0; even-root radicand  $\geq 0$ ; all restrictions must overlap.

## Core graph tools

L01, L03, L04

- Linear functions use slope to show increasing or decreasing direction.
- Quadratic features come from axis, vertex, opening direction, and interval endpoints.
- For a quadratic on a restricted interval, check the vertex and both endpoints.
- Real-world formulas need geometric or physical domain restrictions, not only algebra.

## Composite, equal, and piecewise functions

L02

- For  $f(g(x))$ , the inner value  $g(x)$  must land inside the domain of  $f$ .
- Two functions are equal only when their domains and rules are both the

same.

- Letters are placeholders; domain and rule are what matter.
- Piecewise functions are evaluated from the correct interval first, then substituted.

## Monotonicity and transformations

L04-L05

- Reciprocal functions are monotonic on each side of their break point, not across it.
- Difference quotients can prove increasing or decreasing behavior.
- Inside horizontal shifts act opposite the sign; outside vertical shifts act with the sign.
- For  $f(x+c)$ , pass the whole expression  $x+c$  into the original interval.

## Parity, absolute value, and rational form

L05-L06

- Even function:  $f(-x)=f(x)$ , with  $y$ -axis symmetry.
- Odd function:  $f(-x)=-f(x)$ , with origin symmetry.
- For absolute-value or piecewise functions, decide which branch  $-x$  belongs to before comparing.
- A non-degenerate linear fractional function has a vertical asymptote, horizontal asymptote, and center.

## teacher tips from class

### DOMAIN FIRST, SIMPLIFY SECOND · L01

Before simplifying an expression, write the original restrictions.  
Do not let cancellation accidentally enlarge the domain.

### RANGE FROM PIECES · L03

For a piecewise range question, sketch each interval piece separately.  
Then merge all possible  $y$ -values.

### QUADRATIC INTERVAL ROUTINE · L04

First read the opening direction.  
Then compare the axis of symmetry with the interval endpoints.

### **PARITY ROUTINE · L05-L06**

Check whether the domain is symmetric about 0 before testing  $f(-x)$ .  
For absolute value or piecewise functions, decide the branch first.

### **RATIONAL GRAPH ROUTINE · L06**

Find the two asymptotes and one known point first.  
Exclude degenerate constant cases before claiming monotonic behavior.

### **CHOOSE THE QUADRATIC FORM · L01**

Use factored form when zeros are given.  
Use vertex form when the vertex or axis is given.

### **ENDPOINT NOTATION CHECK · L01**

Use square brackets only when an endpoint is actually included.  
Infinity never gets a square bracket.

### **PARAMETER DEGENERATION FIRST · L01**

In parameter quadratic problems, first check whether the quadratic term vanishes.  
Only after that discuss opening direction, axis, and extrema.

### **DISCRETE INPUT STAYS DISCRETE · L02**

If the domain is a finite or discrete set, graph isolated points.  
Do not connect them just because the formula looks linear.

### **BOUNDS MUST BE REACHED · L03**

A maximum or minimum is not just a bound.  
The function must actually attain that value.

### DO NOT CROSS A BREAK · L04

A monotonic interval cannot cross a point where the function is undefined.

Split intervals at holes, asymptotes, and branch changes.

### AX+B/X SHAPE READ · L06

The sign of  $ab$  decides the double-hook or double-slash shape.

Turning points occur when the two absolute-value parts are equal.

## video example

L01 classroom example

### PROBLEM

Domain of  $\sqrt{1-x} + \sqrt{x+3} - 1$

Step 1: Need  $1-x \geq 0$  and  $x+3 \geq 0$ .

Step 2: So  $x \leq 1$  and  $x \geq -3$ .

Step 3: Domain:  $[-3, 1]$ .

## video example

L02 example

### PROBLEM

If  $f$  has domain  $[0,1]$ , find the domain of  $f(\sqrt{x}-2)$

Step 1: Require  $\sqrt{x}-2$  in  $[0,1]$ .

Step 2: So  $0 \leq \sqrt{x}-2 \leq 1$ .

Step 3: Then  $2 \leq \sqrt{x} \leq 3$ , giving  $4 \leq x \leq 9$ .

## video example

L06 example

### PROBLEM

Show  $f(x) = x|x| + px$  is odd

Step 1: Remove absolute value by branches.

Step 2: For  $x > 0$ , compare  $f(x)$  with  $f(-x)$  on the negative branch.

Step 3: The result is  $f(-x) = -f(x)$ , so the function is odd.