MCR3U

Grade 11 Functions — Practice Workbook

Ontario Grade 11 Functions (MCR3U)

10 lessons • 144 multiple-choice • 54 short-answer questions

Work through each lesson's questions, then check the Answer Key at the back. Pairs with the interactive Grade 11 Study Hub.

Student notebook handouts

The video-distilled functions notebooks are linked here for quick printing or review.

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Unit 1 Functions: handwritten | visual notebook

Unit 2 Exponential Functions: handwritten | visual notebook

Unit 3 Trigonometry: handwritten | visual notebook

Unit 4 Sequences & Series: handwritten | visual notebook

1.1 Transformations

Key formula / rule: Vertex form: $y = a(x - h)^2 + k$

Big idea: Functions can be transformed in predictable ways.

Multiple choice

1. If the vertex form is $y = (x - 3)^2 + 2$, what is the vertex?
A. (3, 2)
B. (2, 3)
C. (-3, 2)
D. (0, 0)

2. How does the graph of $y = -2(x + 1)^2 + 5$ look?
A. Opens up, vertex (1, 5)
B. Opens up, vertex (−1, 5)
C. Opens down, vertex (1, −5)
D. Opens down, vertex (−1, 5)

3. What is the vertex of $y = (x - 5)^2 - 3$?
A. (5, 3)
B. (−5, 3)
C. (5, −3)
D. (−5, −3)

4. What is the vertex of $y = 2(x + 4)^2 + 1$?
A. (−4, −1)
B. (4, 1)
C. (−4, 1)
D. (4, −1)

5. Which equation opens downward?
A. $y = 0.5x^2 - 4$
B. $y = -3(x - 1)^2 + 2$
C. $y = 4x^2 + 1$
D. $y = (x + 2)^2$

6. Compared to $y = x^2$, the graph of $y = (x - 2)^2 + 5$ is shifted…
A. right 2, down 5
B. left 2, up 5
C. left 2, down 5
D. right 2, up 5

7. Describe the graph of $y = x^2 - 6$.
A. Vertex (0, −6), opens up
B. Vertex (0, −6), opens down
C. Vertex (−6, 0), opens down
D. Vertex (0, 6), opens up

8. Compared to $y = x^2$, the graph of $y = 3x^2$ is…
A. narrower (vertical stretch)
B. wider
C. shifted up

D. shifted right

9. Compared to $y = x^2$, the graph of $y = \frac{1}{2}x^2$ is…
A. reflected
B. narrower
C. wider
D. shifted down

10. What is the equation of $y = x^2$ reflected in the x-axis?
A. $y = x^2 + 1$
B. $y = x^2 - 1$
C. $y = (-x)^2$
D. $y = -x^2$

11. For $y = -(x - 3)^2 + 4$, what is the maximum value?
A. 3
B. -4
C. no maximum
D. 4

12. Write the vertex form for a parabola with vertex $(-2, 7)$ and $a = 1$.
A. $y = (x - 2)^2 + 7$
B. $y = (x + 2)^2 - 7$
C. $y = (x - 7)^2 + 2$
D. $y = (x + 2)^2 + 7$

Short answer

1. Explain in words how a, h, and k each change the graph of $y = a(x - h)^2 + k$.

2. Describe the graph of $y = -2(x - 1)^2 + 3$: direction, vertex, and maximum or minimum value.

3. Write the equation of a parabola with vertex $(4, -5)$ that opens up with the same shape as $y = x^2$.

4. The graph of $y = x^2$ is reflected in the x-axis and then shifted up 6. Write the new equation.

1.2 Key Features

Key formula / rule: For $y = a(x - h)^2 + k$: vertex = (h, k), axis of symmetry is $x = h$.
Big idea: A graph tells a story about the function.

Multiple choice

1. For $y = (x + 1)^2 - 4$, what is the axis of symmetry?
A. $x = 1$
B. $x = -1$
C. $y = -4$
D. $x = 4$

2. For $y = 2(x - 3)^2 + 1$, what is the range?
A. all real numbers
B. $y \leq 1$
C. $y \geq 3$
D. $y \geq 1$

3. What is the axis of symmetry of $y = 2(x - 4)^2 + 1$?
A. $x = -4$
B. $y = 1$
C. $x = 1$
D. $x = 4$

4. What is the y-intercept of $y = (x - 1)^2 + 2$?
A. (0, 1)
B. (0, 3)
C. (0, −3)
D. (0, 2)

5. What is the range of $y = -(x - 2)^2 + 5$?
A. $y \leq 2$
B. $y \leq 5$
C. $y \geq 5$
D. $y \geq -5$

6. What is the domain of any quadratic function?
A. Only the vertex
B. $x \leq 0$
C. All real numbers
D. $x \geq 0$

7. How many x-intercepts does $y = (x - 3)^2 - 4$ have?
A. 1
B. 3
C. 2
D. 0

8. How many x-intercepts does $y = (x - 3)^2 + 4$ have?
A. 0
B. 3
C. 1
D. 2

9. Is the vertex of $y = (x + 5)^2 - 2$ a maximum or minimum?

A. Maximum
B. Minimum
C. Both
D. Neither

10. Find the x-intercepts of $y = 2(x - 1)^2 - 8$.
A. No real intercepts
B. $x = 3$ and $x = -1$
C. $x = 1$ and $x = -1$
D. $x = 2$ and $x = -2$

11. The axis of symmetry always passes through the…
A. x-intercept
B. vertex
C. origin
D. y-intercept

12. Does $y = -2(x + 1)^2 + 3$ open up or down?
A. Up
B. Down
C. Neither
D. Sideways

13. What is the domain of $\sqrt{(1 - x)} + \sqrt{(x + 3)} - 1$?
A. $x \geq 1$
B. all real numbers
C. $-3 \leq x \leq 1$
D. $x \leq -3$ or $x \geq 1$

14. If f has domain [0, 1], what is the domain of $f(\sqrt{x} - 2)$?
A. $x \geq 0$ only
B. $0 \leq x \leq 1$
C. $4 \leq x \leq 9$
D. $-2 \leq x \leq -1$

15. Why is $f(x) = x|x| + px$ odd for every constant p?
A. $f(-x) = f(x)$
B. $f(-x) = -f(x)$
C. its graph is always above the x-axis
D. it has no absolute value

Short answer

1. For $y = (x - 2)^2 - 9$, find the vertex, axis of symmetry, direction, and both intercepts.

2. Explain why every quadratic has a domain of all real numbers but a restricted range.

3. How can you tell from vertex form how many x-intercepts a parabola has?

4. State the range of $y = -3(x + 2)^2 + 7$.

5. Find the domain of $\sqrt{(1 - x)} + \sqrt{(x + 3)} - 1$, showing the restriction step.

6. If f has domain [0,1], find the domain of $f(\sqrt{x} - 2)$.

7. Show that $f(x) = x|x| + px$ is odd.

2.1 Growth & Decay

Key formula / rule: Exponential form: y = a · b^x

Big idea: Repeated multiplication leads to exponential change.

Multiple choice

1. Which value of b gives exponential decay?
A. b = 1.8
B. b = 0.75
C. b = 2
D. b = 1.1

2. In y = 5(1.2)^x, what is the initial value (the value at x = 0)?
A. 6
B. 0
C. 5
D. 1.2

3. In y = 3 · 2^x, what is the initial value?
A. 3
B. 2
C. 6
D. 1

4. Is y = 5(0.9)^x growth or decay?
A. Growth
B. Decay
C. Constant

D. Neither

5. Evaluate y = 2(3)^x at x = 2.
A. 6
B. 12
C. 18
D. 8

6. Evaluate y = 100(0.5)^x at x = 3.
A. 50
B. 12.5
C. 25
D. 0.125

7. Which base gives exponential growth?
A. 1
B. 1.5
C. 0.2
D. 0.8

8. The y-intercept of y = a · b^x is always at…
A. (a, 0)
B. (0, b)
C. (0, 0)
D. (0, a)

9. For decay (0 < b < 1), as x gets large, y approaches…
A. 1
B. a negative number
C. 0
D. infinity

10. For y = 4(2)^x, the values at x = 0 and x = 1 are…
A. 4 then 6
B. 4 then 8
C. 2 then 4
D. 8 then 16

11. A decay factor of 0.85 means each step the amount…
A. drops to 15%
B. doubles
C. keeps 85% (drops 15%)
D. grows by 85%

12. A growth factor of 1.2 means a percent increase of…
A. 2%
B. 20%
C. 120%
D. 12%

13. Simplify $\sqrt[4]{(3-\pi)^4}$.
A. $3-\pi$
B. $(3-\pi)^2$

C. $\pi - 3$
D. $\pi + 3$

14. If $x + 1/x = 3$, what is $x^2 + 1/x^2$?
A. 11
B. 7
C. 9
D. 5

15. When simplifying an even root like $\sqrt{(a^2)}$, the teacher move is to use…
A. absolute value
B. the reciprocal
C. no restriction
D. a negative sign only

Short answer

1. Explain the roles of a and b in $y = a \cdot b^x$.

2. For $y = 200(0.9)^x$, state the initial value, whether it grows or decays, and the percent change per step.

3. Describe how the graphs of $y = 2^x$ and $y = (½)^x$ differ.

4. Why does an exponential decay graph never reach $y = 0$?

5. Simplify $\sqrt[4]{((3 - \pi)^4)}$ and explain the sign.

6. If $x + 1/x = 3$, find $x^2 + 1/x^2$.

2.2 Exponential Modelling

Key formula / rule: Model form: $N = a(1 \pm r)^t$
Big idea: A good model connects the equation to the context words.

Multiple choice

1. In N = 500(1.08)^t, what does 500 represent?
A. The ending amount
B. The starting amount
C. The growth rate
D. The time

2. A $200 investment grows 5% per year. Which model fits?
A. N = 200(5)^t
B. N = 200(0.95)^t
C. N = 200(0.05)^t
D. N = 200(1.05)^t

3. $1000 grows at 6% per year. Which model fits?
A. N = 1000(1.06)^t
B. N = 1000(6)^t
C. N = 1000(0.94)^t
D. N = 1000(0.06)^t

4. A population of 800 falls 4% per year. Which model fits?
A. N = 800(1.04)^t
B. N = 800(4)^t
C. N = 800(0.04)^t
D. N = 800(0.96)^t

5. Evaluate N = 200(1.10)^t after 1 year.
A. 220
B. 2000
C. 210
D. 100

6. In N = 500(1.08)^t, what does the 1.08 tell you?
A. 8% decay per year
B. the starting amount
C. 8% growth per year
D. the number of years

7. A $20 000 car loses 15% per year. Its value after 1 year is…
A. $23 000
B. $17 000
C. $3 000
D. $5 000

8. A growth rate of 3% gives a factor of…
A. 1.03
B. 0.03
C. 3
D. 0.97

9. A decay rate of 12% gives a factor of…
A. 0.12
B. 1.12
C. 0.88
D. 12

10. In N = a(1 + r)^t, what is r for a rate of 7%?

A. 0.07
B. 1.07
C. 0.7
D. 7

11. Bacteria start at 50 and double every hour. Which model fits?

A. N = 2(50)^t
B. N = 50(0.5)^t
C. N = 50(1.2)^t
D. N = 50(2)^t

12. In N = a(1 ± r)^t, what is N after 0 periods?

A. 1
B. r
C. a (the starting amount)
D. 0

13. The graph $y = a^x$ passes through (3, π). What is a?

A. $\sqrt[3]{\pi}$
B. 3π
C. π^3
D. π/3

14. For $y = a^x$ passing through (3, π), which description of f(x) is correct?

A. 3 to the power πx
B. πx/3
C. x to the power π/3
D. π to the power x/3

Short answer

1. A town of 5000 grows 3% per year. Write the model and find the population after 2 years.

2. A $400 phone loses 20% of its value per year. Write the model and find its value after 1 year.

3. Explain how to turn a percentage rate into the factor for a growth model and for a decay model.

4. In N = a(1 ± r)^t, what does each letter represent?

5. If $y = a^x$ passes through $(3, \pi)$, find a and write the function.

3.1 Unit Circle

Key formula / rule: Point on unit circle = $(\cos\theta, \sin\theta)$

Big idea: Trig values come from coordinates on a circle of radius 1.

Multiple choice

1. At 180°, what is the point on the unit circle?
A. (0, 1)
B. (-1, 0)
C. (1, 0)
D. (0, -1)

2. At 90°, what is $\sin\theta$?
A. 0.5
B. 0
C. 1
D. -1

3. What is the point on the unit circle at 0°?
A. $(-1, 0)$
B. (0, 1)
C. (1, 0)
D. $(0, -1)$

4. What is the point on the unit circle at 90°?
A. (1, 0)
B. (0, 1)
C. $(0, -1)$
D. $(-1, 0)$

5. What is the point on the unit circle at 270°?
A. $(0, -1)$
B. (0, 1)
C. (1, 0)
D. $(-1, 0)$

6. What is $\cos 180°$?
A. 0.5
B. 1
C. -1
D. 0

7. What is $\sin 0°$?

A. 1
B. −1
C. 0.5
D. 0

8. In which quadrant is an angle of 120°?
A. Quadrant I
B. Quadrant III
C. Quadrant II
D. Quadrant IV

9. In which quadrant is an angle of 200°?
A. Quadrant I
B. Quadrant II
C. Quadrant IV
D. Quadrant III

10. What is the sign of sin θ in Quadrant III?
A. Negative
B. Positive
C. Zero
D. Undefined

11. What is the sign of cos θ in Quadrant II?
A. Negative
B. Zero
C. Positive
D. Undefined

12. At 45°, how do sin θ and cos θ compare?
A. They are equal
B. sin is bigger
C. Both are 0
D. cos is bigger

13. What is sin(2π/3)?
A. √3/2
B. 1/2
C. −√3/2
D. −1/2

14. If tan α = 5/12 and α is in Quadrant III, what are sin α and cos α?
A. sin α = 12/13, cos α = 5/13
B. sin α = −5/13, cos α = −12/13
C. sin α = 5/13, cos α = 12/13
D. sin α = −12/13, cos α = 5/13

15. For trig values, the teacher routine is: reference angle gives the size, and the quadrant gives the…
A. domain only
B. amplitude
C. period
D. sign

Short answer

1. Explain why a point on the unit circle has coordinates (cos θ, sin θ).

2. Give the coordinates of the point at 0°, 90°, 180°, and 270°.

3. How do the signs of sin θ and cos θ change from quadrant to quadrant?

4. At 45°, why are sin θ and cos θ equal?

5. Evaluate sin(2π/3) using quadrant and reference-angle language.

6. Given tan α = 5/12 in Quadrant III, find sin α and cos α.

3.2 Sine Graphs

Key formula / rule: $y = A \sin((360/P)(x - D)) + C$

Big idea: Trig graphs are controlled by a small number of important parameters.

Multiple choice

1. For $y = 3\sin(x) + 1$, what is the midline?
   A. $y = -1$
   B. $y = 1$
   C. $y = 0$
   D. $y = 3$

2. In $y = A \sin((360/P)(x - D)) + C$, what does P stand for?
   A. The phase shift
   B. The midline
   C. The amplitude
   D. The period

3. What is the amplitude of $y = 4 \sin x$?
   A. 0
   B. -4

C. 1
D. 4

4. What is the midline of y = sin x + 3?
A. y = 1
B. y = 0
C. y = −3
D. y = 3

5. What is the period of y = sin((360/120)x)?
A. 360°
B. 60°
C. 240°
D. 120°

6. What is the maximum value of y = 2 sin x + 1?
A. −1
B. 3
C. 2
D. 1

7. What is the minimum value of y = 2 sin x + 1?
A. −3
B. 1
C. −1
D. 0

8. What is the range of y = 3 sin x?
A. 0 to 6
B. −3 to 3
C. 0 to 3
D. −1 to 1

9. The amplitude is the distance from the midline to the…
A. origin
B. y-axis
C. x-axis
D. maximum

10. What is the midline of y = sin x − 2?
A. y = −2
B. y = −1
C. y = 2
D. y = 0

11. If the period is 180°, the coefficient 360/P equals…
A. 360
B. 2
C. 0.5
D. 180

12. The vertical shift C moves the graph…
A. wider or narrower

B. up or down
C. it does nothing
D. left or right

13. For $y = 3\sin(2x + \pi/3) - 1$, what are the amplitude and period?
A. Amplitude 3, period π
B. Amplitude 2, period 3π
C. Amplitude 1, period $\pi/3$
D. Amplitude 3, period 2π

14. In $y = 3\sin(2x + \pi/3) - 1$, the midline is…
A. $y = -1$
B. $y = 2$
C. $y = \pi/3$
D. $y = 3$

15. When solving extrema or monotonic intervals for a transformed sine graph, first solve the…
A. domain of a square root
B. outside amplitude only
C. y-intercept only
D. inside phase

Short answer

1. For $y = 3 \sin x + 2$, state the amplitude, midline, maximum, and minimum.

2. Explain how to find the period of $y = A \sin((360/P)(x - D)) + C$.

3. Describe what each of A, P, D, and C does to a sine graph.

4. A sine curve has a maximum of 7 and a minimum of 1. Find the amplitude and midline.

5. Find the amplitude, period, and midline of $y = 3\sin(2x + \pi/3) - 1$.

3.3 Solving Triangles

Key formula / rule: Cosine law: $a^2 = b^2 + c^2 - 2bc \cos(A)$

Big idea: Non-right triangles need tools beyond SOHCAHTOA.

Multiple choice

1. Which formula is correct when solving side a with sides b, c and included angle A?
A. $a^2 = b^2 + c^2 - 2bc\cos(A)$
B. $a^2 = b^2 - c^2$
C. $a^2 = b^2 + c^2 + 2bc\cos(A)$
D. $a = b + c$

2. When should you reach for the cosine law?
A. Only for right triangles
B. Given two sides and the angle between them (SAS)
C. Given one side only
D. Given two angles only

3. Which is the cosine law for side a?
A. $a^2 = b^2 + c^2 - 2bc\cos(A)$
B. $a = b + c$
C. $a^2 = b^2 + c^2 + 2bc\cos(A)$
D. $a^2 = b^2 - c^2$

4. When can you use the cosine law?
A. Two angles only
B. One side only
C. Right triangles only
D. SAS or SSS

5. For b = 5, c = 7, A = 90°, find a (since cos 90° = 0).
A. ≈ 74
B. ≈ 8.6
C. ≈ 12
D. ≈ 3.5

6. To find an angle when you know all three sides, you…
A. cannot solve it
B. use the sine law
C. rearrange the cosine law
D. use SOHCAHTOA

7. The included angle is the angle…
A. that is always 90°
B. between the two known sides
C. at the top of the triangle
D. opposite the longest side

8. If A = 60°, what is cos A?
A. 1
B. 0.87
C. 0.5
D. 0

9. When A = 90°, the cosine law reduces to…
A. $a^2 = b^2 - c^2$

B. $a = \sqrt{(bc)}$
C. $a^2 = b^2 + c^2$ (Pythagoras)
D. $a = b + c$

10. Which piece is NOT needed to use the cosine law in the SAS case?
A. side b
B. side c
C. angle A
D. the third angle

11. The cosine law solves for the side that is…
A. always the longest
B. opposite the known angle
C. always the shortest
D. adjacent to the known angle

12. For angles in degrees, your calculator must be in…
A. gradian mode
B. radian mode
C. degree mode
D. any mode

13. Using the cosine law with $b = 5$, $c = 7$, and $A = 60°$, what is a^2?
A. $\sqrt{39}$
B. 39
C. 35
D. 74

14. For the cosine-law example $b = 5$, $c = 7$, $A = 60°$, what is side a?
A. $\sqrt{39}$
B. 39
C. $\sqrt{74}$
D. 6

Short answer

1. State the cosine law and explain when to use it.

2. Given $b = 6$, $c = 10$, and $A = 60°$, set up and solve for side a.

3. Explain how the cosine law reduces to the Pythagorean theorem.

4. You know all three sides of a triangle. Describe how to find one of the angles.

5. Use the cosine law to find side a when b = 5, c = 7, and A = 60°.

4.1 Arithmetic Sequences

Key formula / rule: $a_n = a_1 + (n - 1)d$

Big idea: An arithmetic sequence adds the same amount every step.

Multiple choice

1. If $a_1 = 4$ and $d = 3$, what is the 5th term?

A. 19
B. 13
C. 15
D. 16

2. An arithmetic sequence has $a_1 = 10$ and $d = -2$. What is a_4?

A. 4
B. 2
C. 6
D. 8

3. If $a_1 = 2$ and $d = 5$, what is a_3?

A. 7
B. 15
C. 10
D. 12

4. If $a_1 = 10$ and $d = -3$, what is a_4?

A. −2
B. 1
C. 7
D. 4

5. What is the common difference of 3, 7, 11, 15, ...?

A. 4
B. 2
C. 7
D. 3

6. If $a_1 = 0$ and $d = 6$, what is a_{10}?

A. 66
B. 54
C. 48
D. 60

7. Is 2, 4, 8, 16 an arithmetic sequence?

A. Yes, d = 2
B. Yes, d = 4
C. No (it is geometric)
D. Yes, d = 8

8. What is the explicit formula for an arithmetic sequence?
A. $a_n = a_1 \cdot r^{(n - 1)}$
B. $a_n = a_1 + nd$
C. $a_n = a_1 - d$
D. $a_n = a_1 + (n - 1)d$

9. For $a_1 = 5$ and $d = 2$, which term equals 19?
A. 9th
B. 8th
C. 7th
D. 14th

10. A negative common difference means the sequence…
A. stays constant
B. increases
C. alternates
D. decreases

11. If $a_1 = -4$ and $d = 3$, what is a_5?
A. -1
B. 8
C. 5
D. 11

12. The graph of an arithmetic sequence is…
A. a smooth curve
B. discrete points along a line
C. a circle
D. a parabola

13. In an arithmetic sequence, why does $a_1 + a_8 = a_4 + a_5$?
A. All arithmetic sequences are constant
B. The terms must all be zero
C. The index sums are equal: 1 + 8 = 4 + 5
D. The common difference is always 1

14. If $S_n = 2n^2 - 5n$, what is a_1?
A. -3
B. 4
C. -7
D. 3

15. If $S_n = 2n^2 - 5n$, what is a_n?
A. $2n^2 - 5n$
B. $4n - 7$
C. $2n - 5$
D. $4n + 7$

Short answer

1. State the explicit formula for an arithmetic sequence and define each variable.

2. For 7, 11, 15, 19, … find d and the 10th term.

3. Explain how to check whether a sequence is arithmetic.

4. The 3rd term of an arithmetic sequence is 12 and d = 5. Find the first term.

5. Explain why $a_1 + a_8 = a_4 + a_5$ in an arithmetic sequence.

6. If $S_n = 2n^2 - 5n$, find a_n.

4.2 Geometric Sequences

Key formula / rule: $a_n = a_1r^{(n - 1)}$

Big idea: Geometric sequences model repeated multiplication.

Multiple choice

1. If $a_1 = 3$ and r = 2, what is the 4th term?
   A. 18
   B. 24
   C. 27
   D. 12

2. What is the common ratio of 2, 6, 18, 54, …?
   A. 2
   B. 4
   C. 6
   D. 3

3. If $a_1 = 2$ and r = 3, what is a_3?
   A. 8
   B. 18

C. 6
D. 12

4. What is the common ratio of 4, 8, 16, 32, …?
A. 4
B. 0.5
C. 2
D. 8

5. If $a_1 = 5$ and $r = 2$, what is a_4?
A. 20
B. 40
C. 25
D. 80

6. What is the common ratio of 81, 27, 9, 3, …?
A. 9
B. -3
C. 3
D. 1/3

7. If r is between 0 and 1, the terms…
A. shrink (decay)
B. stay the same
C. alternate sign
D. grow without bound

8. If $a_1 = 1$ and $r = 2$, what is a_5?
A. 8
B. 16
C. 32
D. 10

9. Is 3, 6, 9, 12 a geometric sequence?
A. No (it is arithmetic)
B. Yes, r = 1.5
C. Yes, r = 2
D. Yes, r = 3

10. If $a_1 = 100$ and $r = 0.5$, what is a_3?
A. 12.5
B. 25
C. 50
D. 75

11. What is the explicit formula for a geometric sequence?
A. $a_n = a_1 + r^n$
B. $a_n = a_1 \cdot r^{(n - 1)}$
C. $a_n = a_1 + (n - 1)d$
D. $a_n = a_1 \cdot r \cdot n$

12. If $a_1 = 2$ and $a_2 = 10$, what is the common ratio?
A. 5

B. 10
C. 2
D. 8

13. Given $(n + 1)a_{n+1} = na_n$ and $a_1 = 1$, what is a_n?
A. n
B. n + 1
C. 2^n
D. 1/n

14. The recurrence $(n + 1)a_{n+1} = na_n$ is best attacked by…
A. turning it into a product of ratios
B. drawing a sine graph
C. using the cosine law
D. finding a vertical asymptote

15. Before using shifted subtraction on a geometric-style sum, what special case should be checked?
A. $q = \pi$
B. n is even only
C. $q = 1$
D. $a_1 = 0$ only

Short answer

1. State the explicit formula for a geometric sequence and define each variable.

2. For 3, 6, 12, 24, … find r and the 6th term.

3. Explain how to tell an arithmetic sequence from a geometric one.

4. The 2nd term of a geometric sequence is 10 and $r = 2$. Find a_1 and a_4.

5. Solve $(n + 1)a_{n+1} = na_n$ with $a_1 = 1$.

4.3 Mixed Review

Key formula / rule: Review focus: identify the function type, then apply the matching language and formula.

Big idea: Strong function sense means recognizing patterns across different representations.

Multiple choice

1. Which function type is most directly associated with repeated percentage change?
A. Arithmetic sequence
B. Exponential function
C. Constant relation
D. Linear relation

2. A pattern goes 5, 8, 11, 14, ... What type is it?
A. Geometric sequence
B. Arithmetic sequence
C. Quadratic function
D. Exponential function

3. The pattern 2, 5, 8, 11 is...
A. geometric
B. arithmetic
C. exponential
D. quadratic

4. The pattern 3, 6, 12, 24 is...
A. geometric
B. linear
C. arithmetic
D. constant

5. Repeated percentage change is best modelled by a...
A. arithmetic sequence
B. exponential function
C. linear relation
D. constant relation

6. The form $y = a(x - h)^2 + k$ is the...
A. arithmetic rule
B. vertex form of a quadratic
C. sine graph
D. exponential model

7. The pair $(\cos \theta, \sin \theta)$ describes a point on the...
A. number line
B. parabola
C. bar graph
D. unit circle

8. The relationship $a^2 = b^2 + c^2 - 2bc \cos(A)$ is the...
A. cosine law
B. exponential model
C. quadratic formula
D. sine law

9. Which has a common difference?
A. exponential function

B. unit circle
C. geometric sequence
D. arithmetic sequence

10. Which has a common ratio?
A. geometric sequence
B. quadratic
C. arithmetic sequence
D. linear relation

11. Midline, amplitude, and period describe which graphs?
A. straight lines
B. sine / trigonometric graphs
C. parabolas
D. bar charts

12. What is the domain of every quadratic function?
A. $x \geq 0$
B. the vertex only
C. all real numbers
D. only integers

13. Which teacher move appears across many distilled math examples?
A. Ignore domain restrictions
B. Identify the structure before choosing a formula
C. Guess from the answer choices first
D. Always expand everything

14. Which pair is correctly matched?
A. Even root → drop absolute value
B. Trig quadrant → ignore signs
C. S_n to a_n → use $a_n = S_n - S_{n-1}$
D. Cosine law → only right triangles

Short answer

1. List the four function families covered in this course and one identifying feature of each.

2. Describe your step-by-step approach to an unfamiliar function question.

3. Explain the difference between arithmetic and geometric sequences, with an example of each.

4. How do you decide whether a real-world situation should be modelled as exponential?

5. List three distilled math teacher moves and give a one-line use case for each.

Answer Key

MCR3U — Functions

1.1 Transformations

Multiple choice

1. A ((3, 2)) — In $y = a(x - h)^2 + k$ the vertex is (h, k). The bracket $(x - 3)$ gives $h = 3$, and the $+2$ gives $k = 2$, so the vertex is (3, 2).

2. D (Opens down, vertex (−1, 5)) — $a = -2$ is negative, so the parabola opens down. Rewrite $(x + 1)$ as $(x - (-1))$, so $h = -1$ and $k = 5$. The vertex is (−1, 5).

3. C ((5, −3)) — In $y = a(x - h)^2 + k$ the vertex is (h, k). Here $h = 5$ and $k = -3$, so the vertex is (5, −3).

4. C ((−4, 1)) — Rewrite $(x + 4)$ as $(x - (-4))$, so $h = -4$ and $k = 1$. The vertex is (−4, 1).

5. B ($y = -3(x - 1)^2 + 2$) — A parabola opens down when a is negative. Only $y = -3(x - 1)^2 + 2$ has a negative a.

6. D (right 2, up 5) — $h = 2$ shifts right 2, and $k = 5$ shifts up 5.

7. A (Vertex (0, −6), opens up) — $h = 0$ and $k = -6$ give vertex (0, −6); $a = 1 > 0$ so it opens up.

8. A (narrower (vertical stretch)) — A larger $|a|$ stretches the parabola vertically, making it narrower.

9. C (wider) — When $0 < |a| < 1$ the parabola is compressed vertically, making it wider.

10. D ($y = -x^2$) — Reflecting in the x-axis multiplies the output by −1, giving $y = -x^2$.

11. D (4) — $a < 0$ so it opens down; the vertex (3, 4) is the highest point, so the maximum value is 4.

12. D ($y = (x + 2)^2 + 7$) — Vertex (h, k) = (−2, 7) gives $y = (x - (-2))^2 + 7 = (x + 2)^2 + 7$.

Short answer

1. a controls the vertical stretch/compression and, if negative, reflects the graph (changes shape and direction). h shifts the graph left/right ($h > 0$ moves right). k shifts it up/down ($k > 0$ moves up).

2. $a = -2 < 0$ so it opens down. Vertex (1, 3). Because it opens down, the vertex is a maximum, so the maximum value is 3 (at $x = 1$); axis of symmetry $x = 1$.

3. $y = (x - 4)^2 - 5$ ($a = 1$ keeps the same shape, $h = 4$, $k = -5$).

4. $y = -x^2 + 6$ (reflection makes a negative, then $+6$ shifts up).

1.2 Key Features

Multiple choice

1. B (x = -1) — Rewrite $(x + 1)$ as $(x - (-1))$, so $h = -1$. The axis of symmetry is the vertical line $x = h$, which is $x = -1$.

2. D ($y \geq 1$) — $a = 2 > 0$, so the parabola opens up and the vertex (3, 1) is the lowest point. The y-values start at 1 and go up, so the range is $y \geq 1$.

3. D ($x = 4$) — The axis of symmetry is $x = h$. Here $h = 4$, so it is $x = 4$.

4. B ((0, 3)) — Set $x = 0$: $(0 - 1)^2 + 2 = 1 + 2 = 3$, so the y-intercept is (0, 3).

5. B ($y \le 5$) — It opens down with vertex (2, 5) as the maximum, so y values go down from 5: $y \le 5$.

6. C (All real numbers) — Parabolas extend left and right forever, so the domain is all real numbers.

7. C (2) — The vertex (3, −4) is below the x-axis and it opens up, so it crosses the x-axis twice.

8. A (0) — The vertex (3, 4) is above the x-axis and it opens up, so it never reaches the x-axis.

9. B (Minimum) — $a = 1 > 0$ so it opens up; the vertex is the lowest point, a minimum.

10. B ($x = 3$ and $x = -1$) — Set $y = 0$: $2(x - 1)^2 = 8$, so $(x - 1)^2 = 4$, $x - 1 = \pm 2$, giving $x = 3$ or $x = -1$.

11. B (vertex) — The axis of symmetry is the vertical line $x = h$ that runs through the vertex.

12. B (Down) — $a = -2$ is negative, so the parabola opens down.

13. C ($-3 \le x \le 1$) — Both radicands must be non-negative: $1 - x \ge 0$ and $x + 3 \ge 0$, so $x \le 1$ and $x \ge -3$.

14. C ($4 \le x \le 9$) — Require $\sqrt{x} - 2$ to land in [0,1]. Then $0 \le \sqrt{x} - 2 \le 1$, so $2 \le \sqrt{x} \le 3$ and $4 \le x \le 9$.

15. B ($f(-x) = -f(x)$) — $f(-x) = (-x)|-x| + p(-x) = -x|x| - px = -(x|x| + px)$.

Short answer

1. Vertex (2, −9); axis $x = 2$; opens up. y-intercept: $x = 0 \to (0 - 2)^2 - 9 = -5$, so (0, −5). x-intercepts: $(x - 2)^2 = 9 \to x - 2 = \pm 3 \to x = 5$ or $x = -1$, so (5, 0) and (−1, 0).

2. A parabola continues left and right forever, so x can be anything (domain = all reals). But it turns at the vertex, so y only reaches values above the vertex (opens up) or below it (opens down) — the range is bounded by k.

3. Compare the vertex height k with the opening direction. Opens up with $k < 0$ (or down with $k > 0$): 2 intercepts. $k = 0$: exactly 1 (vertex on the axis). Opens up with $k > 0$ (or down with $k < 0$): 0 real intercepts.

4. $a = -3 < 0$ opens down, vertex (−2, 7) is the maximum, so the range is $y \le 7$.

5. Need $1 - x \ge 0$ and $x + 3 \ge 0$. So $x \le 1$ and $x \ge -3$. Domain: $-3 \le x \le 1$.

6. The input $\sqrt{x} - 2$ must be in [0,1]. So $0 \le \sqrt{x} - 2 \le 1 \to 2 \le \sqrt{x} \le 3 \to 4 \le x \le 9$.

7. $f(-x) = (-x)|-x| + p(-x) = -x|x| - px = -(x|x| + px) = -f(x)$. Therefore f is odd.

2.1 Growth & Decay

Multiple choice

1. B ($b = 0.75$) — Decay needs a base between 0 and 1, so each step multiplies by less than 1. Only $b = 0.75$ satisfies $0 < b < 1$.

2. C (5) — At $x = 0$, $b^0 = 1$, so $y = a \cdot 1 = a$. The initial value is the coefficient $a = 5$.

3. A (3) — At $x = 0$, $2^0 = 1$, so $y = 3$. The initial value is the coefficient $a = 3$.

4. B (Decay) — The base 0.9 is between 0 and 1, so the function decays.

5. C (18) — $y = 2 \cdot 3^2 = 2 \cdot 9 = 18$.

6. B (12.5) — $y = 100 \cdot 0.5^3 = 100 \cdot 0.125 = 12.5$.

7. B (1.5) — Growth needs a base greater than 1. Only 1.5 qualifies.

8. D ((0, a)) — At $x = 0$, $b^0 = 1$, so $y = a$. The y-intercept is (0, a).

9. C (0) — Repeated multiplication by a number less than 1 shrinks y toward 0.

10. B (4 then 8) — At x = 0, y = 4; at x = 1, y = 4 · 2 = 8. The value doubles each step.

11. C (keeps 85% (drops 15%)) — Multiplying by 0.85 keeps 85% of the previous amount, a 15% decrease.

12. B (20%) — 1.2 = 1 + 0.2, so the increase is 0.2 = 20% per step.

13. C (π − 3) — An even root gives an absolute value: $\sqrt[4]{((3-\pi)^4)} = |3-\pi|$. Since π > 3, this is π − 3.

14. B (7) — $(x + 1/x)^2 = x^2 + 2 + 1/x^2$. So $x^2 + 1/x^2 = 3^2 - 2 = 7$.

15. A (absolute value) — Even roots return the non-negative value, so $\sqrt{(a^2)} = |a|$.

Short answer

1. a is the initial value — the y-intercept, the value when x = 0. b is the growth/decay factor multiplied each step: b > 1 gives growth, 0 < b < 1 gives decay.

2. Initial value 200. It decays (b = 0.9 < 1). Each step keeps 90%, a 10% decrease per step.

3. y = 2^x grows (rises to the right); y = (½)^x decays (falls to the right). Both pass through (0, 1) and are mirror images of each other across the y-axis.

4. A positive starting value multiplied by a positive factor stays positive forever. The curve approaches the x-axis as an asymptote but never actually equals 0.

5. Because the root index is even, $\sqrt[4]{((3-\pi)^4)} = |3-\pi|$. Since π > 3, $|3-\pi| = \pi - 3$.

6. Square both sides: $(x + 1/x)^2 = x^2 + 2 + 1/x^2 = 9$. Therefore $x^2 + 1/x^2 = 7$.

2.2 Exponential Modelling

Multiple choice

1. B (The starting amount) — N = a(1 + r)^t. The coefficient a is the value when t = 0, so 500 is the starting amount.

2. D (N = 200(1.05)^t) — Growth of 5% means the factor is 1 + 0.05 = 1.05. With starting amount 200, the model is N = 200(1.05)^t.

3. A (N = 1000(1.06)^t) — Growth factor = 1 + 0.06 = 1.06, with starting amount 1000.

4. D (N = 800(0.96)^t) — Decay factor = 1 − 0.04 = 0.96, with starting amount 800.

5. A (220) — N = 200 · 1.10 = 220.

6. C (8% growth per year) — 1.08 = 1 + 0.08, an 8% increase each year.

7. B ($17 000) — N = 20000 · (1 − 0.15) = 20000 · 0.85 = 17 000.

8. A (1.03) — Growth factor = 1 + 0.03 = 1.03.

9. C (0.88) — Decay factor = 1 − 0.12 = 0.88.

10. A (0.07) — Write the percent as a decimal: 7% = 0.07.

11. D (N = 50(2)^t) — Doubling means a factor of 2, with starting amount 50.

12. C (a (the starting amount)) — At t = 0 the factor is raised to the power 0, which is 1, so N = a.

13. A ($\sqrt[3]{\pi}$) — Substitute the point: $a^3 = \pi$, so a is the cube root of π.

14. D (π to the power x/3) — Since $a = \sqrt[3]{\pi}$, $a^x = (\sqrt[3]{\pi})^x$ = π to the power x/3.

Short answer

1. N = 5000(1.03)^t. After t = 2: $5000(1.03)^2 = 5000(1.0609) \approx 5305$ people.

2. N = 400(0.80)^t. After 1 year: 400(0.80) = $320.

3. Write the rate as a decimal r. Growth factor = 1 + r; decay factor = 1 − r. For example 7% → r = 0.07, growth factor 1.07.

4. a = starting amount; r = rate per period (as a decimal); t = number of periods; use + for growth and − for decay.

5. Substitute (3, π): $a^3 = \pi$, so $a = \sqrt[3]{\pi}$. Therefore $f(x) = (\sqrt[3]{\pi})^x$, equivalently π to the power x/3.

3.1 Unit Circle

Multiple choice

1. B ((-1, 0)) — The point is (cos θ, sin θ). At 180° you are on the negative x-axis, so cos 180° = −1 and sin 180° = 0, giving (−1, 0).

2. C (1) — At 90° the point on the unit circle is (0, 1). Since y = sin θ, sin 90° = 1.

3. C ((1, 0)) — At 0° you are on the positive x-axis: (cos 0°, sin 0°) = (1, 0).

4. B ((0, 1)) — At 90° you are at the top: (cos 90°, sin 90°) = (0, 1).

5. A ((0, −1)) — At 270° you are at the bottom: (cos 270°, sin 270°) = (0, −1).

6. C (−1) — At 180° the point is (−1, 0), and x = cos θ, so cos 180° = −1.

7. D (0) — At 0° the point is (1, 0), and y = sin θ, so sin 0° = 0.

8. C (Quadrant II) — Angles between 90° and 180° lie in Quadrant II.

9. D (Quadrant III) — Angles between 180° and 270° lie in Quadrant III.

10. A (Negative) — In Quadrant III the y-coordinate is negative, and y = sin θ, so sin θ is negative.

11. A (Negative) — In Quadrant II the x-coordinate is negative, and x = cos θ, so cos θ is negative.

12. A (They are equal) — At 45° the point is (√2/2, √2/2), so cos 45° = sin 45°.

13. A (√3/2) — 2π/3 is in Quadrant II, where sine is positive. The reference angle is π/3, so sin(2π/3) = √3/2.

14. B (sin α = −5/13, cos α = −12/13) — Use the 5-12-13 triangle. In Quadrant III, sine and cosine are both negative while tangent is positive.

15. D (sign) — Reduce to a reference angle for the absolute value, then use the quadrant to attach the correct sign.

Short answer

1. On a circle of radius r, cos θ = x/r and sin θ = y/r. The unit circle has r = 1, so x = cos θ and y = sin θ.

2. 0° → (1, 0); 90° → (0, 1); 180° → (−1, 0); 270° → (0, −1).

3. QI: both positive. QII: cos negative, sin positive. QIII: both negative. QIV: cos positive, sin negative.

4. 45° lies on the line y = x, where the point is (√2/2, √2/2). Equal coordinates mean cos 45° = sin 45°.

5. 2π/3 is in Quadrant II. The reference angle is π/3. Sine is positive in Quadrant II, so sin(2π/3) = √3/2.

6. Use a 5-12-13 triangle. In Quadrant III, both x and y are negative, so sin α = −5/13 and cos α = −12/13.

3.2 Sine Graphs

Multiple choice

1. B (y = 1) — The midline is y = C, the vertical shift. Here C = +1, so the midline is y = 1. (The 3 is the amplitude.)

2. D (The period) — P is the period — how many degrees the wave takes to complete one full cycle. A is amplitude, C is the midline, D is the phase shift.

3. D (4) — Amplitude is |A|. Here A = 4, so the amplitude is 4.

4. D (y = 3) — The midline is y = C, the vertical shift. Here C = 3.

5. D (120°) — In the 360/P form, P is the period. Here P = 120, so the period is 120°.

6. B (3) — Max = midline + amplitude = 1 + 2 = 3.

7. C (−1) — Min = midline − amplitude = 1 − 2 = −1.

8. B (−3 to 3) — Amplitude 3 with midline 0 gives values from −3 to 3.

9. D (maximum) — Amplitude measures from the midline up to a maximum (or down to a minimum).

10. A (y = −2) — The vertical shift is C = −2, so the midline is y = −2.

11. B (2) — 360 ÷ 180 = 2, so the coefficient of x is 2.

12. B (up or down) — C raises or lowers the whole curve, shifting the midline up or down.

13. A (Amplitude 3, period π) — Amplitude is |3| = 3. Period is 2π/|2| = π.

14. A (y = −1) — The outside vertical shift is −1, so the midline is y = −1.

15. D (inside phase) — The distilled teacher move is to set the inside expression as the phase u, use the base sine rule, then solve for x.

Short answer

1. Amplitude 3; midline y = 2; maximum = 2 + 3 = 5; minimum = 2 − 3 = −1.

2. In this form P is the period directly (in degrees). Equivalently, period = 360 ÷ |the coefficient of x|.

3. A = amplitude (distance from midline to a max/min); P = period (length of one full cycle); D = phase/horizontal shift; C = vertical shift (the midline).

4. Amplitude = (7 − 1)/2 = 3; midline = (7 + 1)/2 = 4 (y = 4).

5. Amplitude = |3| = 3. Period = 2π/|2| = π. Midline is y = −1.

3.3 Solving Triangles

Multiple choice

1. A ($a^2 = b^2 + c^2 - 2bc\cos(A)$) — The cosine law subtracts the angle term: $a^2 = b^2 + c^2 - 2bc\cos(A)$. The minus sign is what makes it work for non-right triangles.

2. B (Given two sides and the angle between them (SAS)) — The cosine law works for SAS (two sides plus the included angle) or SSS (all three sides). With two angles you would use the sine law instead.

3. A ($a^2 = b^2 + c^2 - 2bc\cos(A)$) — The cosine law subtracts the angle term: $a^2 = b^2 + c^2 - 2bc\cos(A)$.

4. D (SAS or SSS) — It works given two sides and the included angle (SAS) or all three sides (SSS).

5. B (≈ 8.6) — $a^2 = 25 + 49 - 0 = 74$, so $a = \sqrt{74} \approx 8.6$.

6. C (rearrange the cosine law) — With SSS you rearrange the cosine law to solve for the cosine of an angle.

7. B (between the two known sides) — The included angle sits between the two sides you know.

8. C (0.5) — cos 60° = 0.5, a standard special-angle value.

9. C ($a^2 = b^2 + c^2$ (Pythagoras)) — cos 90° = 0, so the $-2bc \cos A$ term vanishes, leaving $a^2 = b^2 + c^2$.

10. D (the third angle) — SAS only needs two sides and the angle between them; the other angles are not required.

11. B (opposite the known angle) — Side a is opposite angle A — the side across from the included angle.

12. C (degree mode) — If the angle is in degrees, the calculator must be set to degree mode.

13. B (39) — $a^2 = b^2 + c^2 - 2bc \cos A = 25 + 49 - 2(5)(7)(1/2) = 39$.

14. A ($\sqrt{39}$) — The calculation gives $a^2 = 39$, so $a = \sqrt{39}$.

Short answer

1. $a^2 = b^2 + c^2 - 2bc \cos A$. Use it for SAS (two sides and the included angle) or SSS (three sides, to find an angle).

2. $a^2 = 6^2 + 10^2 - 2(6)(10)\cos 60° = 36 + 100 - 120(0.5) = 136 - 60 = 76$, so $a = \sqrt{76} \approx 8.7$.

3. When A = 90°, cos 90° = 0, so the $-2bc \cos A$ term disappears, leaving $a^2 = b^2 + c^2$.

4. Rearrange the cosine law: $\cos A = (b^2 + c^2 - a^2) / (2bc)$, then take $A = \cos^{-1}$ of that value.

5. $a^2 = b^2 + c^2 - 2bc \cos A = 25 + 49 - 2(5)(7)(1/2) = 74 - 35 = 39$, so $a = \sqrt{39}$.

4.1 Arithmetic Sequences

Multiple choice

1. D (16) — Use $a_n = a_1 + (n - 1)d$. $a_5 = 4 + (5 - 1)(3) = 4 + 12 = 16$.

2. A (4) — $a_4 = a_1 + (4 - 1)d = 10 + 3(-2) = 10 - 6 = 4$. A negative d means the sequence decreases.

3. D (12) — $a_3 = 2 + (3 - 1)(5) = 2 + 10 = 12$.

4. B (1) — $a_4 = 10 + (4 - 1)(-3) = 10 - 9 = 1$.

5. A (4) — Subtract consecutive terms: 7 − 3 = 4. The common difference is 4.

6. B (54) — $a_{10} = 0 + (10 - 1)(6) = 9 \cdot 6 = 54$.

7. C (No (it is geometric)) — The terms multiply by 2 each time (a common ratio), not add a fixed amount, so it is geometric.

8. D ($a_n = a_1 + (n - 1)d$) — The explicit rule is $a_n = a_1 + (n - 1)d$.

9. B (8th) — $5 + (n - 1)(2) = 19 \rightarrow n - 1 = 7 \rightarrow n = 8$.

10. D (decreases) — Adding a negative number each step makes the terms get smaller.

11. B (8) — $a_5 = -4 + (5 - 1)(3) = -4 + 12 = 8$.

12. B (discrete points along a line) — Sequences are defined only for whole-number term positions, so the graph is separate points lying on a straight line.

13. C (The index sums are equal: 1 + 8 = 4 + 5) — Equal index sums cross the same total amount of common difference, so the paired term sums match.

14. A (−3) — $a_1 = S_1 = 2(1)^2 - 5(1) = -3$.

15. B ($4n - 7$) — For $n \geq 2$, $a_n = S_n - S_{n-1}$. This simplifies to $4n - 7$, and it also works for n = 1.

Short answer

1. $a_n = a_1 + (n - 1)d$, where a_n is the nth term, a_1 is the first term, n is the term number, and d is the common difference.

2. $d = 4$ and $a_1 = 7$. $a_{10} = 7 + (10 - 1)(4) = 7 + 36 = 43$.

3. Find the difference between consecutive terms. If the same number is added every time (constant difference), it is arithmetic.

4. $a_3 = a_1 + 2d = 12 \rightarrow a_1 = 12 - 2(5) = 12 - 10 = 2$.

5. The index sums are both 9. In an arithmetic sequence, terms with equal index sums have equal paired sums because the common-difference shifts balance.

6. First $a_1 = S_1 = -3$. For $n \geq 2$, $a_n = S_n - S_{n-1} = (2n^2 - 5n) - [2(n - 1)^2 - 5(n - 1)] = 4n - 7$. This also gives $a_1 = -3$.

4.2 Geometric Sequences

Multiple choice

1. B (24) — Use $a_n = a_1 r^{(n - 1)}$. $a_4 = 3(2)^{(4 - 1)} = 3 \times 8 = 24$.

2. D (3) — Divide each term by the one before it: $6 \div 2 = 3$ and $18 \div 6 = 3$. The common ratio r is 3.

3. B (18) — $a_3 = 2 \cdot 3^{(3 - 1)} = 2 \cdot 9 = 18$.

4. C (2) — Divide consecutive terms: $8 \div 4 = 2$. The common ratio is 2.

5. B (40) — $a_4 = 5 \cdot 2^{(4 - 1)} = 5 \cdot 8 = 40$.

6. D (1/3) — $27 \div 81 = 1/3$. Each term is one-third of the previous.

7. A (shrink (decay)) — Multiplying by a number less than 1 each step makes the terms smaller.

8. B (16) — $a_5 = 1 \cdot 2^{(5 - 1)} = 2^4 = 16$.

9. A (No (it is arithmetic)) — The terms add 3 each time (common difference), so it is arithmetic, not geometric.

10. B (25) — $a_3 = 100 \cdot 0.5^{(3 - 1)} = 100 \cdot 0.25 = 25$.

11. B ($a_n = a_1 \cdot r^{(n - 1)}$) — The explicit rule is $a_n = a_1 r^{(n - 1)}$.

12. A (5) — $r = a_2 \div a_1 = 10 \div 2 = 5$.

13. D (1/n) — Rewrite as $a_{n+1}/a_n = n/(n + 1)$. Multiplying the ratios telescopes, giving $a_n = 1/n$.

14. A (turning it into a product of ratios) — This is a multiplicative recurrence, so divide by a_n and multiply the ratios to telescope.

15. C ($q = 1$) — The teacher warning is to check $q = 1$ before dividing by $1 - q$.

Short answer

1. $a_n = a_1 r^{(n - 1)}$, where a_n is the nth term, a_1 is the first term, r is the common ratio, and n is the term number.

2. $r = 2$ and $a_1 = 3$. $a_6 = 3 \cdot 2^{(6 - 1)} = 3 \cdot 32 = 96$.

3. Arithmetic adds the same difference each step; geometric multiplies by the same ratio. Check whether consecutive terms differ by a constant amount or a constant factor.

4. $a_1 = a_2 \div r = 10 \div 2 = 5$. $a_4 = 5 \cdot 2^{(4 - 1)} = 5 \cdot 8 = 40$.

5. Rewrite as $a_{n+1}/a_n = n/(n + 1)$. Multiplying from 1 to $n - 1$ cancels middle factors: $a_n/a_1 = 1/n$. Since $a_1 = 1$, $a_n = 1/n$.

4.3 Mixed Review

Multiple choice

1. B (Exponential function) — Repeated percentage change means multiplying by the same factor again and again — that is the signature of an exponential function.

2. B (Arithmetic sequence) — Each term increases by the same amount (+3 each time), so it is an arithmetic sequence with common difference d = 3.

3. B (arithmetic) — It adds 3 each time (common difference), so it is arithmetic.

4. A (geometric) — It multiplies by 2 each time (common ratio), so it is geometric.

5. B (exponential function) — Multiplying by the same factor repeatedly is exponential behaviour.

6. B (vertex form of a quadratic) — That is the vertex form of a quadratic (parabola).

7. D (unit circle) — On the unit circle the coordinates are (cos θ, sin θ).

8. A (cosine law) — That is the cosine law for triangles.

9. D (arithmetic sequence) — A common difference (added each step) defines an arithmetic sequence.

10. A (geometric sequence) — A common ratio (multiplied each step) defines a geometric sequence.

11. B (sine / trigonometric graphs) — Those words describe the features of sine and other trig graphs.

12. C (all real numbers) — Parabolas extend forever left and right, so the domain is all real numbers.

13. B (Identify the structure before choosing a formula) — The examples repeatedly classify the structure first: domain restriction, even root, phase, recurrence, or sequence sum.

14. C (S_n to a_n → use $a_n = S_n - S_{n-1}$) — When a sum formula S_n is given, recover terms with $a_n = S_n - S_{n-1}$ for $n \geq 2$, while checking a_1 separately.

Short answer

1. Quadratic (parabola/vertex form), exponential (constant ratio / percent change), trigonometric (periodic — amplitude & period), and discrete sequences (arithmetic = common difference, geometric = common ratio).

2. Identify the function type → name the parameters and what they mean → connect them to the graph or representation → answer using the correct language and units.

3. Arithmetic adds a constant difference d (e.g., 2, 5, 8, … with d = 3). Geometric multiplies by a constant ratio r (e.g., 2, 6, 18, … with r = 3).

4. Look for repeated multiplication or a constant percent change per period. A constant amount added each period is linear/arithmetic instead.

5. Examples: domain first before simplifying; even roots need absolute value; reference angle gives trig size while quadrant gives sign; S_n to a_n uses $a_n = S_n - S_{n-1}$; multiplicative recurrences can telescope by ratios.